# Probabilistic Spatial Relations for Monitoring Behavior of Road Users

Lennart Siefke[1 2], Volker Sommer[1], Murat Can Baylan[1], Lars Grunske[2]

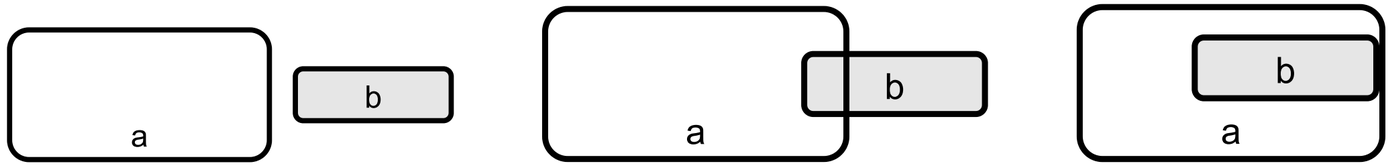
[1] Berlin University of Technology and Applied Sciences

[2] Humboldt University Berlin

# Spatial Relations

- Distance relations
- Neighborhood relations
- Spatio-topological relations

# Spatio-topological Relations




a disjoints b | a overlaps b | a contains b

**Use Cases in Road Traffic**

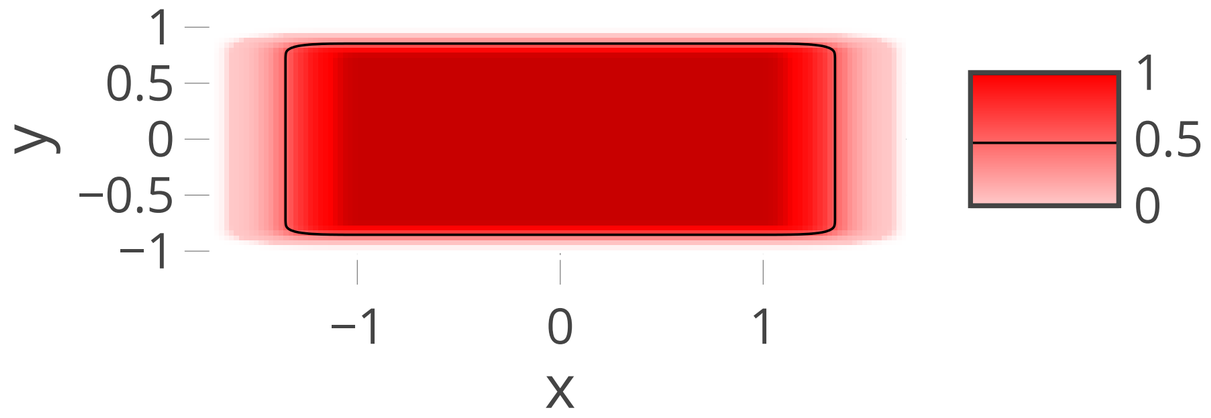
- Will vehicles collide in the nearby future?
- Do the sensors cover the bicycle lane during turning?

# RQ: What to do when poses and/or boundaries are not known without fault?

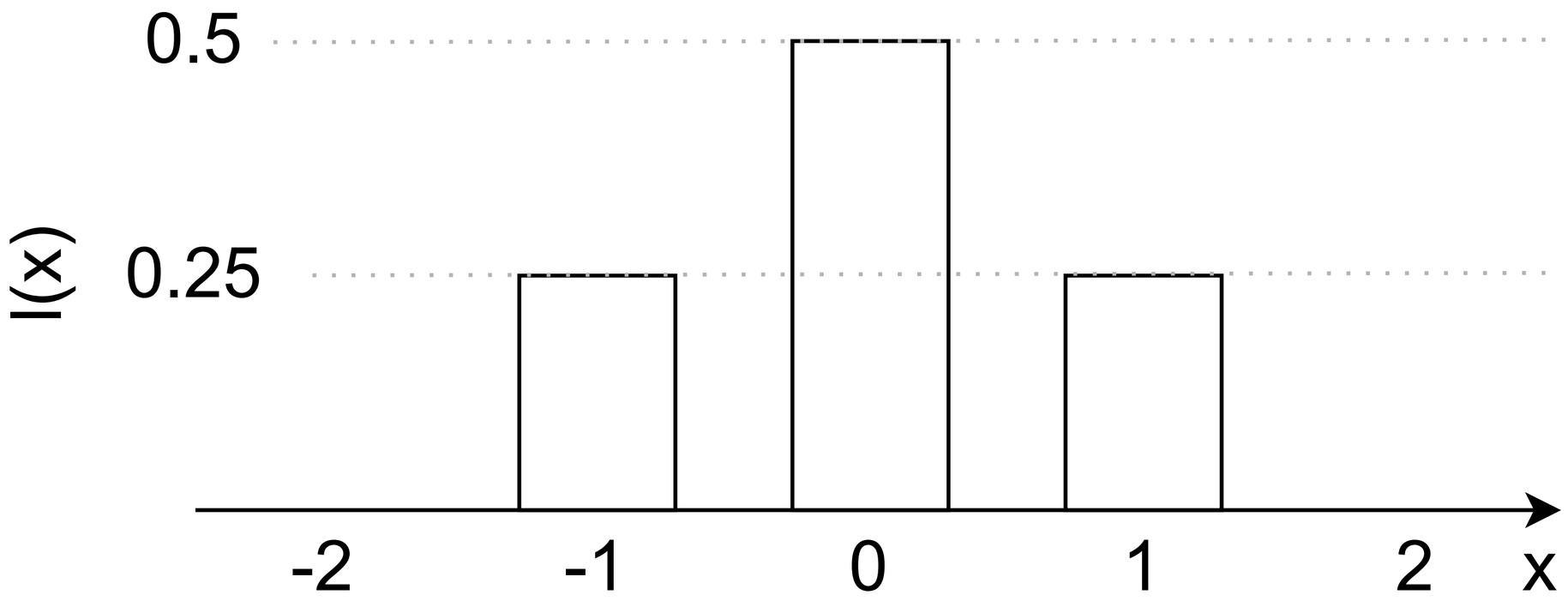
- Pose errors, caused by sensors for example
- Fuzzy boundaries, caused by image segmentation for example

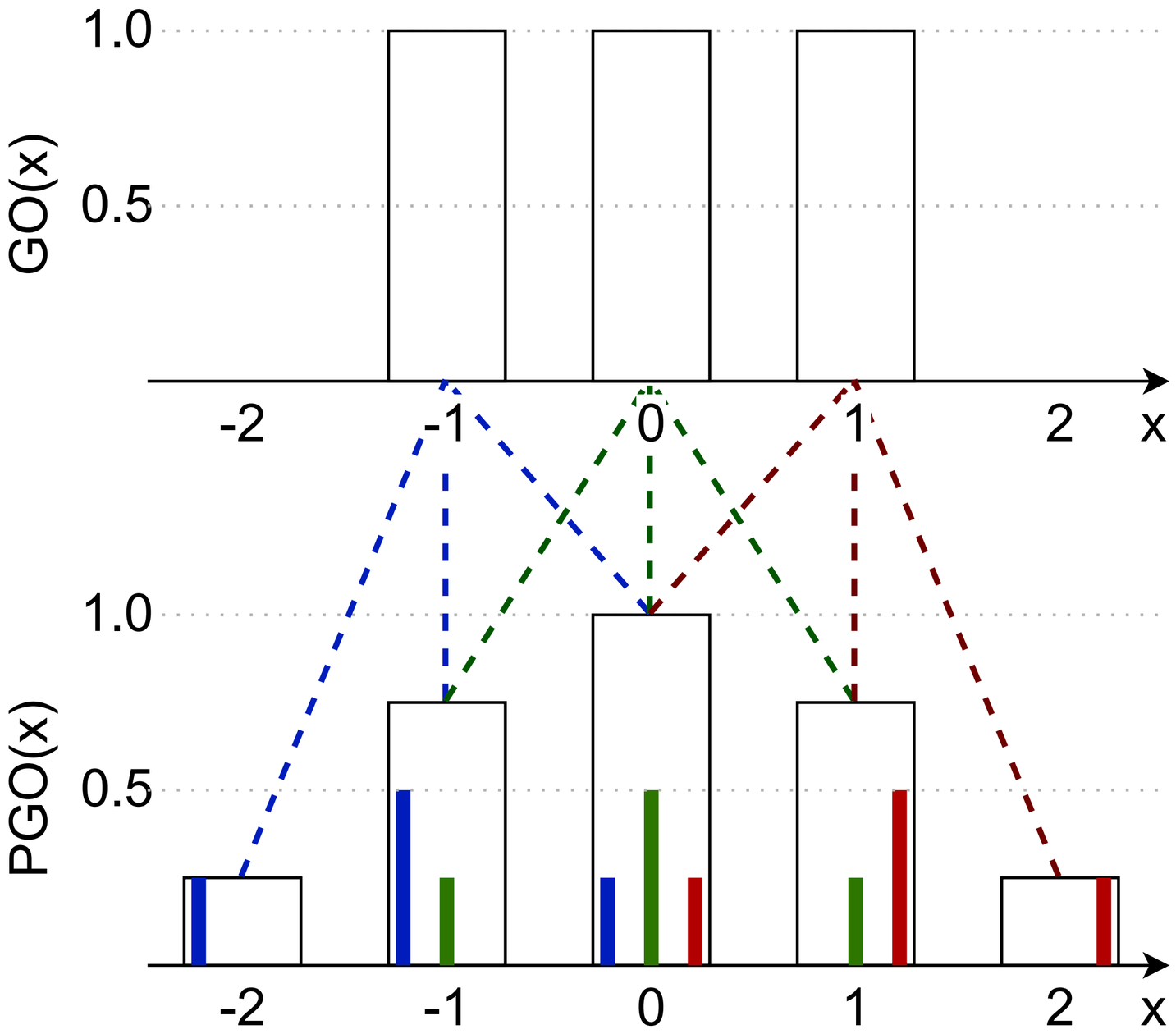
# Probabilistic Geometric Objects

Model of a geometric object, with each point mapped to a probability value indicating occurence of the geometric object at this point.



# Model Pose Errors into Geometric Objects




1.0
0.5
GO(x)
-2
-1
0
1
2
x
1.0
0.5
PGO(x)
-2
-1
0
1
2
x

# Model Pose Errors into Geometric Objects

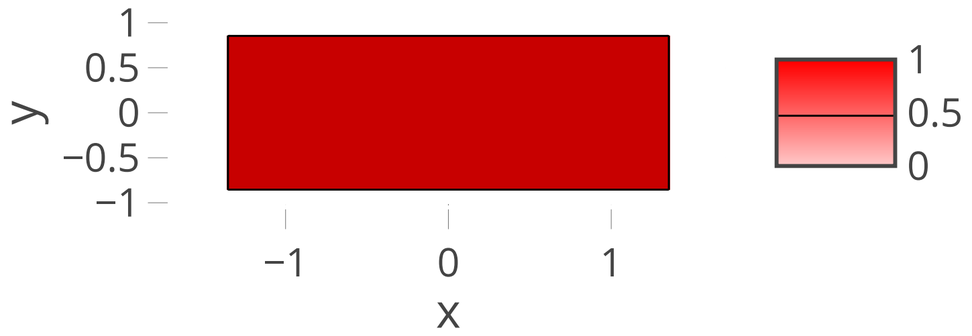
$$PGO(x,y) = \sum_{\tilde{x}=-x_{max}}^{x_{max}} \sum_{\tilde{y}=-y_{max}}^{y_{max}} p(\tilde{x},\tilde{y}) \cdot GO(x-\tilde{x}, y-\tilde{y})$$

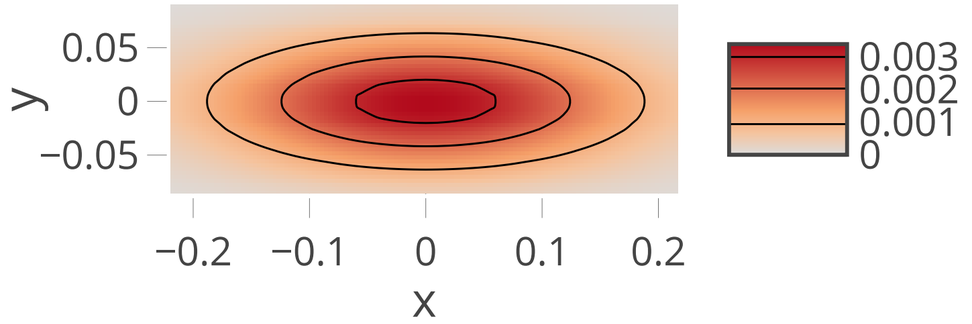
- $GO(x,y)$: Geometric Object
- $PGO(x,y)$: Probabilistic Geometric object
- $p(x,y)$: Discrete probability distribution

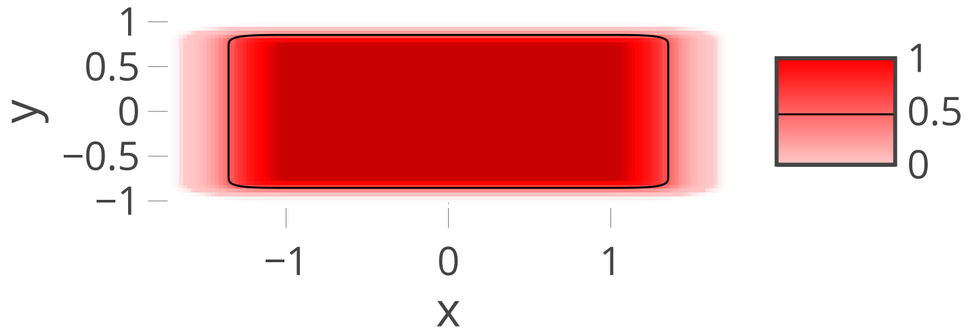
Known as **Convolution**: $PGO(x,y) = p(x,y) * GO(x,y)$

Performance Optimization: $PGO = \mathscr{F}^{-1}(\mathscr{F}(p) \cdot \mathscr{F}(GO))$

# Example


1
0.5
0
−0.5
−1
y
−1
0
1
x
1
0.5
0

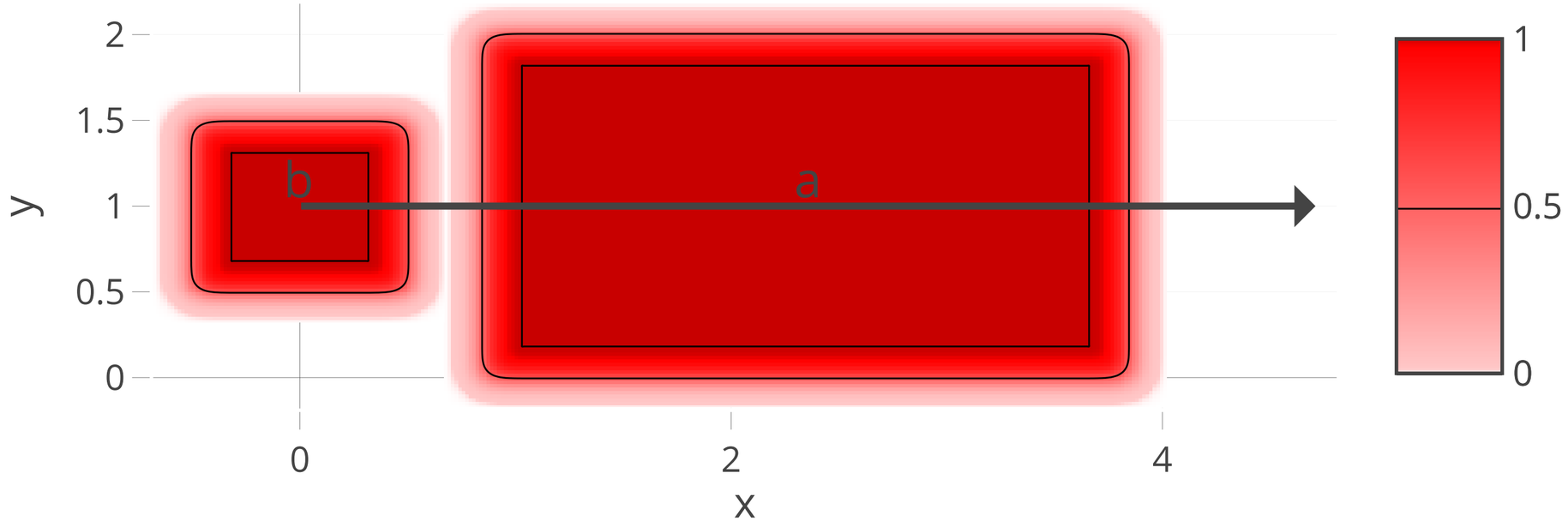

0.05
0
−0.05
y
−0.2
−0.1
0
0.1
0.2
x
0.003
0.002
0.001
0

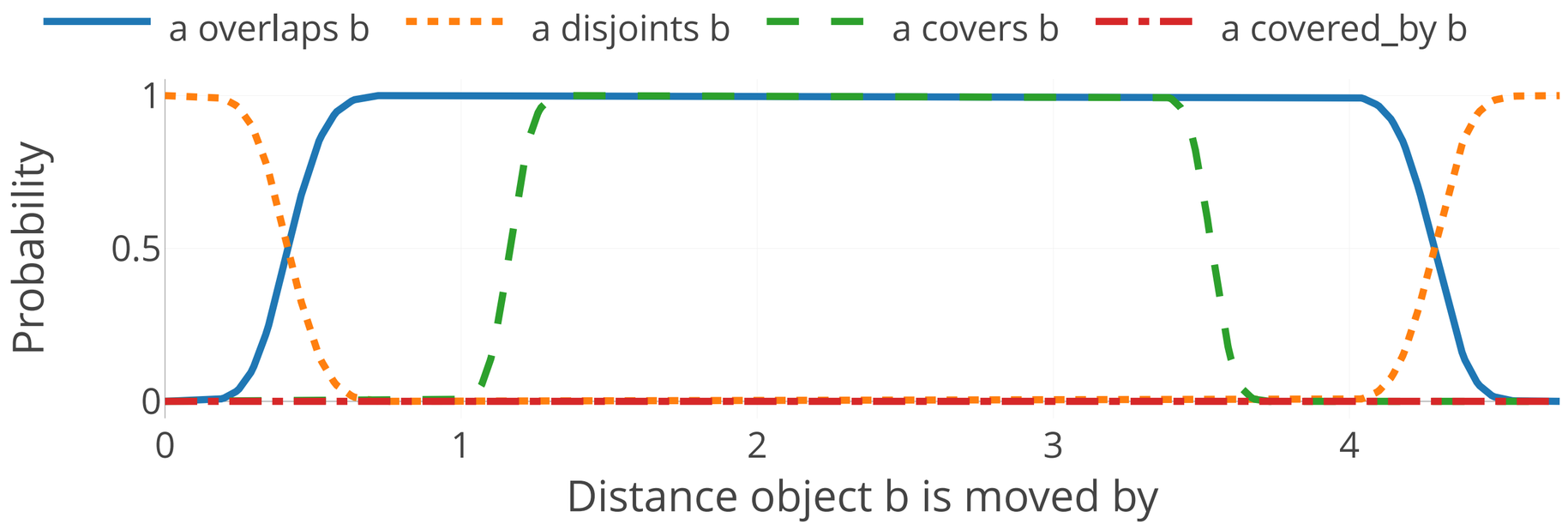

1
0.5
0
−0.5
−1
y
−1
0
1
x
1
0.5
0

# Probabilistic Overlaps

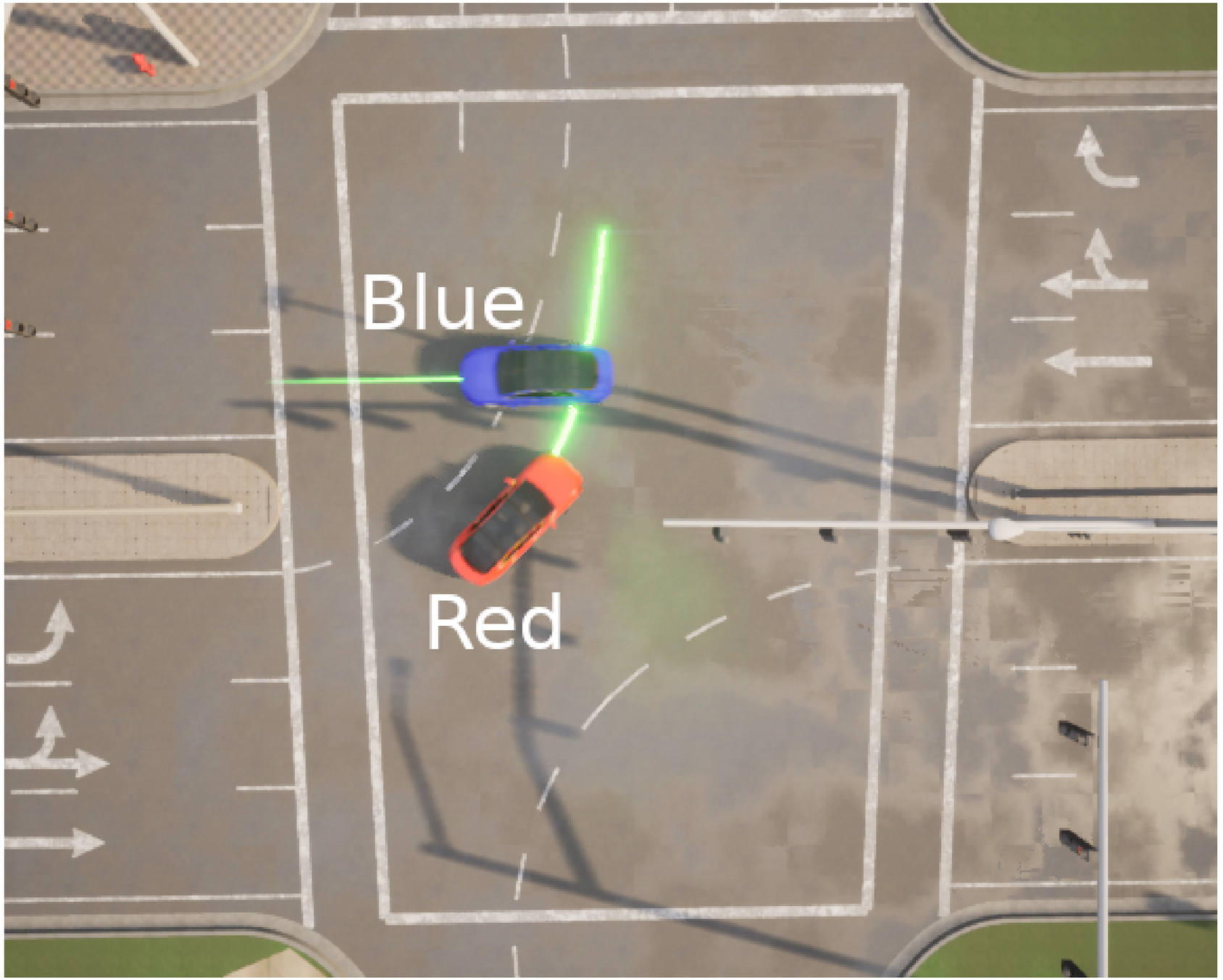
- Probability of $a$ and $b$ overlapping at point $(x|y)$
  - $overlaps_{point}(a, b, (x, y)) = a(x, y) \cdot b(x, y)$
- Probability of $a$ and $b$ overlapping in at least one point in 2D plane
  - $overlaps(a, b) = \max_{(x,y)\in\mathbb{R}^2} a(x, y)b(x, y)$

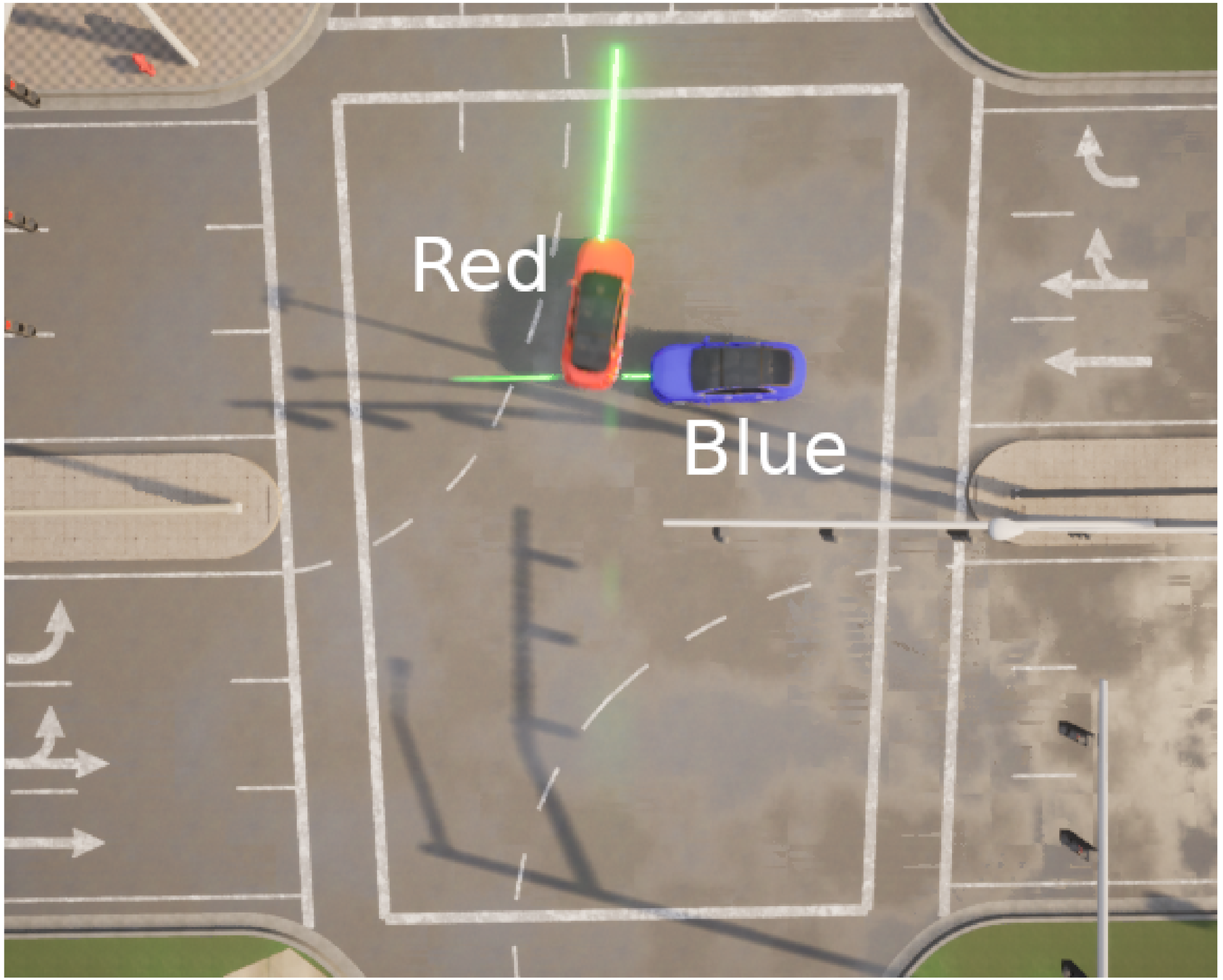
| Operator | Boolean Condition | Probabilistic Condition |
|---|---|---|
| Overlaps | $a \cap b \neq \emptyset$ | $\max_{(x,y)\in\mathbb{R}^2} a(x,y)b(x,y)$ |
| Disjoints | $a \cap b = \emptyset$ | $\min_{(x,y)\in\mathbb{R}^2} 1 - a(x,y)b(x,y)$ |
| Covers | $a \cap b = b$ | $\{\max_{(x,y)\in a\cup b} a(x,y)b(x,y)\} \cdot \{1 - \max_{(x,y)\in b\setminus a} b(x,y)\}$ |
| Covered_by | $a \cap b = a$ | $\{\max_{(x,y)\in a\cup b} a(x,y)b(x,y)\} \cdot \{1 - \max_{(x,y)\in a\setminus b} a(x,y)\}$ |


2
1.5
1
0.5
0
y
b
a
0
2
4
x
1
0.5
0


a overlaps b
a disjoints b
a covers b
a covered_by b
Probability
1
0.5
0
0
1
2
3
4
Distance object b is moved by

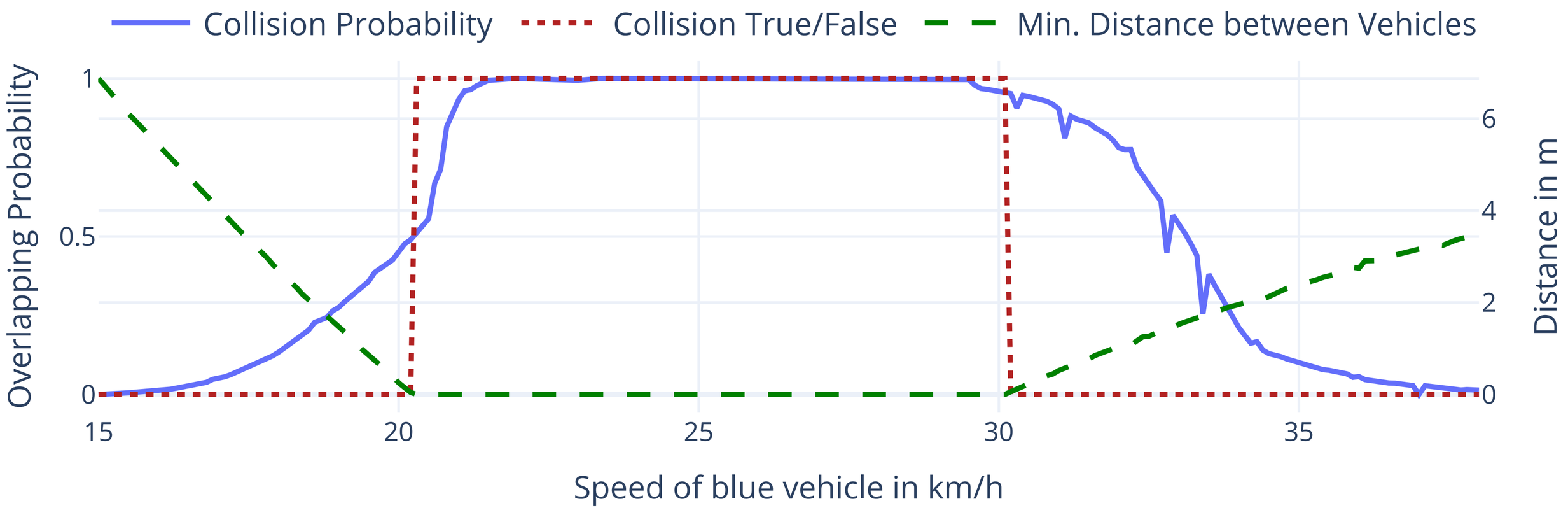
# Experiment with Carla Simulator


Blue
Red


Red
Blue

# Setup of Experiment

- Discrete trajectories are 9m long with poses every 0.5m
- Pose Errors for future points in trajectory:
  - Std. deviation of 0 at presence
  - Std. deviation increases by 0.75 per meter into future along direction of motion
- Temporal monitoring:
  - $collision = max(overlaps_i(red_i, blue_i))$ for $i = [0.5, 1, ..., 9]$
- Evaluation happens every 12.5ms


Collision Probability
Collision True/False
Min. Distance between Vehicles
Overlapping Probability
1
0.5
0
Distance in m
6
4
2
0
15
20
25
30
35
Speed of blue vehicle in km/h

# Conclusion

- Boolean spatio-topological relations were generalized using probabilistics
- Applicability supported by experiment, though more experiments and evaluation would be beneficial
- Next step: Temporal monitoring of probabilistic spatial relations